WEEKLY TEST RANKER'S BATCH TEST - 03 RAJPUR SOLUTION Date 15-09-2019

[PHYSICS]

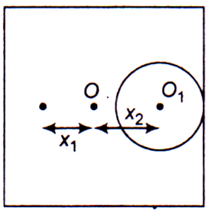
1. Centre of mass of square plate is at O. Centre of mass of two masses of 5 kg each is at H. Hence, centre of mass of the whole system is at mid-point of OH.

2. Distance distributes in inverse ratio of masses. Hence,

$$r_c = d\left(\frac{m_0}{m_0 + m_c}\right) = 1.2 \times 10^{-10}\left(\frac{16}{16+12}\right)$$
$$= 0.69 \times 10^{-10} \text{ m}$$

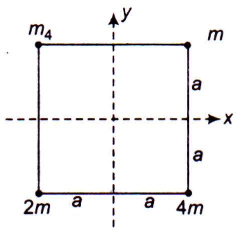
3. $A_1x_1 = A_2x_2$ or $x_1 = \frac{A_2}{A_1} \cdot x_2$

$$= \frac{\left(\frac{\pi}{4}\right)(8)^2}{(20)^2} \times 6 = 0.12 \text{ cm from } O.$$



4. Area of circle, $\frac{\pi}{4}a^2 = A_1$, area of square $= a^2 = A_2$. Since, $A_2 > A_1$ centre of mass will lie inside the square plate.

5. $X_{CM} = 0$



or $\frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = 0$

or $(2m)(-a) + 4m(a) + m(a) + m_4(-a) = 0$

or $m_4 = 3m$

Similarly $y_{CM} = 0$

or $(2m)(-a) + 4m(-a) + m(a) + m_4(a) = 0$

or $m_4 = 5m$

Since, value of m_4 are different to satisfy boy both $x_{CM} = 0$ and $y_{CM} = 0$.

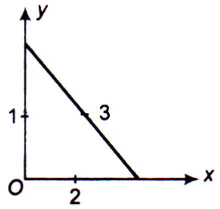
Hence, it is not possible.

6. $$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m(0) + (m)\left(\frac{a}{2}\right) + m\left(\frac{a}{2}\right)}{m + m + m}$$

$$= \frac{a}{3}$$

Similarly $y_{CM} = \frac{a}{3}$



7. Centre of mass of Ist system already lies at (1, 2, 3). Therefore centre of mass of 3 kg and 5 kg should lie at (1, 2, 3).

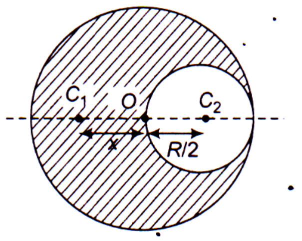
$$\therefore \quad \frac{3(-\hat{i} + 3\hat{j} - 2\hat{k}) + 5\,\mathbf{r}_5}{(3 + 5)} = (\hat{i} + 2\hat{j} + 3\hat{k})$$

Solving this we get, $\mathbf{r}_5 = \frac{11}{5}\hat{i} + \frac{7}{5}\hat{j} + 6\hat{k}$

i.e., 5 kg mass should be kept at (11/5, 7/5, 6)

8. Centre of mass of complete disc should lie at point O. C_1 is the position of centre of mass of remaining portion and C_2 the position of centre of mass of the removed disc.

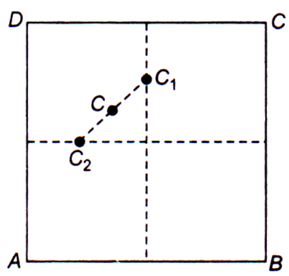
$\therefore$ x (Area of remaining portion) = $\frac{R}{2}$ (Area of removed disc)



$$\therefore \quad x\left[\pi R^2 - \frac{\pi R^2}{4}\right] = \frac{R}{2}\left[\frac{\pi R^2}{4}\right]$$

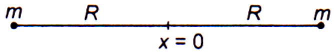
$$\therefore \quad x = \frac{R}{6}$$

9. $C_1 \rightarrow$ Position of centre of mass of rods AB and CD (nearer to CD, as it is heavy)



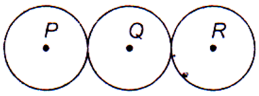
$C_2 \rightarrow$ Position of centre of mass of rods BC and DA.
$C \rightarrow$ Overall centre of mass of all four rods.

10. For a single particle distance of centre of mass from origin is R. For more than one particles distance $\leq R$.



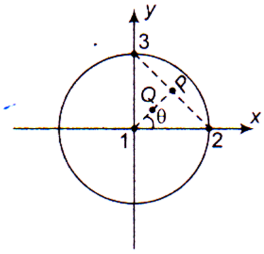
For example for two particles of equal mass, kept as shown in figure, distance = 0.

11. X_{CM} (from P) $= \dfrac{m \times O + m \times PQ + m \times PR}{m + m + m}$ ($m = 1$ kg)



or $$X_{CM} = \frac{PQ + PR}{3}$$

12. P is the position of centre of mass of particle at 2 and 3.
Q is position of centre of mass of all three particles.



$$\tan\theta = \frac{y_{CM}}{x_{CM}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1x_1 + m_2x_2 + m_3x_3}$$
$$= \frac{6 \times 0 + 2 \times 0 + 2 \times a}{6 \times 0 + 2 \times a + 2 \times 0}$$
$$= 1$$
or $$\theta = 45°$$

13. $P_i = 0 \therefore P_f = 0$ or centre of mass should be at rest at all instants.

14. $a_{CM} = \dfrac{(m)(0) + (m)(a)}{m + m} = \dfrac{1}{2}a$

15. $v_{CM} = \dfrac{10(14) + 4(0)}{10 + 14} = 10$ m/s

16. Centre of mass does not change its path during explosion. Therefore it will keep on falling vertically.

17. Net external force is zero. Hence velocity of CM of the box and ball system will remain constant.

18. Both the balls in air have acceleration g downwards. Hence the acceleration of their centre of mass will also be g downwards.

19. Let plank moves x distance in opposite direction. Then, displacement of man relative to ground will be, $(L - x)$. Applying

$$m_R x_R = m_L x_L$$

or $$M(L - x) = \frac{M}{3} x$$

Solving this equation we get, $x = \frac{3L}{4}$

$\therefore$ Displacement of man relative to ground $= L - \frac{3L}{4} = \frac{L}{4}$

20. Centre of mass will remain at height h.

$$\therefore \quad h_{CM} = \frac{m \times 0 + MH}{m + M} = h$$

$$\therefore \quad H = h\left(1 + \frac{m}{M}\right)$$

21. Afer 1 s, coordinates of first particle will become $(4m, 4m, 6m)$ and co-ordinates of second particle will become $(6m, 4m, 8m)$.

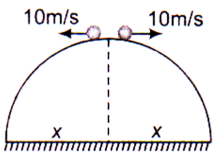
$$\therefore \quad X_{CM} = \frac{4 + 6}{2} = 5\text{m}$$

$$Y_{CM} = \frac{4 + 4}{2} = 4\text{m}$$

and $$Z_{CM} = \frac{6 + 8}{2} = 7\text{m}$$

Note In this problem answer is independent of the fact whether the first particle has velocity v_1 or v_2. Think why?

22. Remaining time for the pieces to reach the ground will be,



$$t = \sqrt{\frac{2 \times 45}{10}} - \sqrt{\frac{2 \times 20}{10}}$$

$$= 1\text{ s}$$

$$x = 10 \times 1 = 10\text{m}$$

$\therefore$ Distance between two pieces will be 20 m.

23. In the absence of friction, centre of mass will not move in x-direction.

24. $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$

$\therefore$ a . b + c.0 = (a + c) v

$$\Rightarrow \quad v = \frac{ab}{a + c}$$

25. At $t = 0$ centre of mass is at mid- point or at (2.25m, 0)

Velocity of centre of mass is zero. Hence centre of mass will remain at this position all the time.

26. K = radius of gyration

$$= \sqrt{\frac{I}{m}} = \sqrt{\frac{\frac{7}{5}mR^2}{m}} = \sqrt{\frac{7}{5}}R$$

27. I_{CM} is less than I about any other axis not passing through center of mass but only when two axes are parallel.

28. Moment of inertia of the rod lying along z-axis will be zero. Of the rods along x and y-axis will be $\frac{ML^2}{3}$ each. Hence, total moment of inertia is $\frac{2}{3}ML^2$.

29. Radius of gyration $K = \sqrt{\frac{I}{m}}$

$$K_{disc} = \sqrt{\frac{\frac{1}{2}mR^2 + mR^2}{m}} = \sqrt{\frac{3}{2}}R$$

$$K_{ring} = \sqrt{\frac{mR^2 + mR^2}{m}} = \sqrt{2}R$$

$$\therefore \quad \frac{K_{disc}}{K_{ring}} = \frac{\sqrt{3/2}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$$

30. $I = I_{CM} + mx^2$

i.e., I-x graph is a parabola not passing through origin.

31. $l = 2\pi R$

$$\therefore \quad R = \frac{l}{2\pi}$$

$$I_1 = \frac{ml^2}{12}$$

$$I_2 = mR^2 = \frac{ml^2}{4\pi^2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{\pi^2}{3}$$

32. Moment of inertia depends on the distribution of mass about the axis.

33. $I = I_{ring} + 3I_{spoke} = MR^2 + 3\left(\frac{mR^2}{3}\right) = (M + m)R^2$

34. $I = \frac{ml^2}{3}$, $I' = \frac{(3m/4)(3l/4)^2}{3}$

$$= \frac{27}{67}\left(\frac{ml^2}{3}\right) = \frac{27}{64}I$$

35. From theorem of parallel axes, $I_2 = I_1 + MR^2$

36. Perpendicular distance from z-axis would be

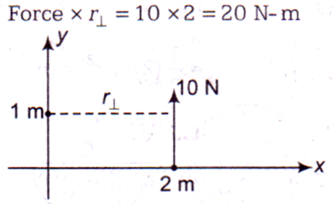
$$\sqrt{(1)^2 + (1)^2} = \sqrt{2}\ m$$

$$\therefore \quad I = mr^2 = (1)(\sqrt{2})^2$$
$$= 2\ \text{kg-m}^2$$

37. $I = 4\left[\frac{M}{2}\frac{(l/2)^2}{3}\right] = \frac{Ml^2}{6}$

38. For square lamina $I_2 = I_3$. This value will be less than I_1 because mass is nearer to axis in this case.

39. Moment = Force $\times r_{\perp} = 10 \times 2 = 20$ N-m



40. The desired ratio is $\dfrac{mR^2}{\frac{1}{2}mR^2} = 2:1$

41. $K = \sqrt{\dfrac{I}{m}} = \sqrt{\dfrac{(mL^2/12)}{m}} = \dfrac{L}{2\sqrt{3}}$

42. For a given body mass is same, so it will depend only on the distribution of mass about the axis.

The mass is farthest from axis BC, so I_2 is maximum. Mass is nearest to axis AC, so I_3 is minimum.

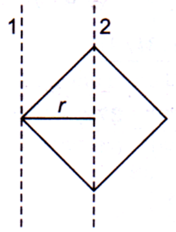
Hence, the correct sequence will be

$$I_2 > I_1 > I_3$$

43. From theorem of perpendicular axes we have

$$I = I_c + m\left(\frac{a}{\sqrt{2}}\right)^2$$
$$= \left[\frac{ma^2}{12} + \frac{ma^2}{12}\right] + \frac{ma^2}{2}$$
$$= \frac{2}{3}ma^2$$

44. This value is independent of angle θ.



$$I_1 = I_2 + (4M)\,r^2$$
$$= 4\left[\frac{ML^2}{3}\sin^2 45°\right] + 4M\left[\frac{\sqrt{2}\,L}{2}\right]^2$$
$$= \frac{8}{3}ML^2$$

$$K_1 = \sqrt{\frac{I_1}{m_1}} = \sqrt{\frac{(5/4)\, m_1 R^2}{m_1}} = \sqrt{\frac{5}{4}}\, R$$

45. 

$$K_2 = \sqrt{\frac{I_2}{m_2}}$$

$$= \sqrt{\frac{(3/2)\, m_2 R^2}{m_2}} = \sqrt{\frac{3}{2}}\, R$$

$$\frac{K_1}{K_2} = \sqrt{\frac{5}{6}}$$

[CHEMISTRY]

46.

$$W = q_V = -nC_V(T_2 - T_1)$$

$$3000 = -1 \times 20 \times (T_2 - 300) \quad \Rightarrow \quad T_2 = \mathbf{150\ K}$$

47.

System is closed and insulated, $Q = 0$ (heat change between system and surrounding). $\Delta E = W + Q = W$ (Since $Q = 0$)

48.

$$q_P = nC_P\Delta T$$

$$\Delta T = \frac{1000}{\left(\frac{100}{18}\right) \times 75} = \mathbf{2.4\ K}$$

49.

Mixture of monoatomic gases will still have monoatomic gases. \

50.

51.

During adiabatic process, no heat is exchanged with surrounding. Hence, $q = 0$.

From $\Delta E = q + W$ (Work done on the system)

$\Delta E = W$ (Since, $q = 0$)

52.

1 Litre-atm = 24.2 calorie

1 calorie = 4.1868 Joule

1 Joule = 10^7 erg

53.

More negative the enthalpy of formation, more is the stability.

54.

$q = 300$ calorie

$W = -P\Delta V = -1 \times 10$ litre-atm $= -10 \times 24.2$ cal $= -242$ cal

$\Delta E = q + W = 300 - 242 =$ **58 cal**

55.

ΔH for isothermal free expansion is zero.

56.

57.

$$\frac{V_2}{V_1} = \frac{1}{10}$$

$$W \text{ (on the system)} = -2.303 nRT \log \frac{V_2}{V_1}$$

$$= -2.303 \times 1 \times 2 \times 500 \log \frac{1}{10} \text{ cal}$$

$$= +\frac{2.303 \times 2 \times 500}{1000} \text{ kcal} = \mathbf{+2.303\ kcal}$$

58.

In cyclic system, $\Delta E = 0$, $\Delta H = 0$.

Work done by the system = − 550 kJ.

$$\Delta E = q + W$$

$$\Rightarrow \quad 0 = q - 550 \quad \Rightarrow \quad \mathbf{q = 550\ kJ}$$

59.

$$W = -2.303 nRT \log \frac{V_2}{V_1}$$

$$= -2.303 \times 2 \times 8.314 \times 300 \times \log \frac{50}{5} \text{ joule}$$

$$= -11488.285 \text{J} \simeq \mathbf{-11.5\ kJ}$$

60.

$$q = +200 \text{J}$$

$$W = -P\Delta V = -1 \times (20 - 10) = -10 \text{ atm L}$$

$$= -10 \times 101.3 \text{ J} = -1013 \text{ J}$$

$$\Delta E = q + W = (200 - 1013) \text{J} = \mathbf{-813\ J}$$

61.

ΔH for isothermal free expansion is zero.

62.

Volume occupied by molecules of a gas can never be zero.

63.

64.

Leakage of a gas from balloon is related with its expansion by taking energy from attractive forces of molecules. This decreases the temperature.

65. In an adiabatic change, no heat is exchanged between the system and the surroundings.

66. : State function

67. Based on the first law of thermodynamics,

$\Delta U = q + w$

Change in internal energy for a cyclic process is zero, *i.e.* $\Delta U = 0$.

$\therefore \quad q = -w$

68. As it absorbs heat, $q = +208$ J

$$w_{rev} = -2.303 nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$w_{rev} = -2.303 \times (0.04) \times 8.314 \times 310 \ \log_{10}\left(\frac{375}{50}\right)$$

$$\therefore \quad w_{rev} = -207.76 \approx -208 \text{ J}$$

69. $T_3 < T_1$ because cooling takes place on adibatic expansion. Hence, (b) is incorrect.

70 $$W = -2.303 nRT \log \frac{V_2}{V_1}$$

$$= -2.303 \times 1 \times 8.314 \times 300 \times \log \frac{20}{10}$$

$= -2.303 \times 8.314 \times 300 \times 0.3010 = -1729$ joules

Work done = – 1729 joules

71. Volume depends on the mass of the system.

72.

73. No work is done along the path AB because this process is isochoric (for isochoric process V = 0

$\therefore$ work done $= PdV = 0$).

Thus, the work done $dw = P_B(V_D - V_A)$

$= 8 \times 10^4 (5 \times 10^{-3} - 2 \times 10^{-3})$

$= 8 \times 10^4 \times 3 \times 10^{-3} J = 240$ J

The energy absorbed by the system

$= (dq)_{AB} + (dq)_{BC} = 600 + 200 = 800$ J

The change in internal energy $dE = dq - dw$

$dE = 800 - 240 = 560$ J

74. $W = -\Delta 2.303\ \Delta nRT\ \log \frac{P_1}{P_2}$

$W = -2.303 \times 1 \times 0.082 \times 300 \text{lof} \frac{1}{10}$

$W = -1381.9$ cal

75.

76.

$$W_{\text{expansion}} = -P\Delta V$$

$$= -(1 \times 10^5 \text{Nm}^{-2})[(1 \times 10^{-2} - 1 \times 10^{-3})\text{m}^3]$$

$$= -10^5 \times (10 \times 10^{-3} - 1 \times 10^{-3})\text{Nm}$$

$$= -10^5 \times 9 \times 10^{-3} \text{J} = -9 \times 10^2 \text{J} = \mathbf{-900\ J}$$

77.

78.

$W_{rev} > W_{irrev}$; Thus, there will be more cooling in reversible process.

79.

80.

$q = +40.65 \text{ kJ mol}^{-1}$

$W_{exp.} = -3.1$ kJ

$\Delta E = q + W$

$= 40.65 - 3.1 = \mathbf{37.55\ kJ}$

81.

As the system starts from A and reaches to A again, whatever the stages may be net energy change is **zero**.

82.

83

84

85. (c) During isothermal expansion of an ideal gas against vacuum is zero because expansion is isothermal. The reason, that volume occupied by the molecules of an ideal gas is zero, is false.

86. (a) it is fact that absolute values of internal energy of substances can not be determined. It is also true that to determine exact values of constituent energies of the substance is impossible.

87. (b) Mass and volume are extensive properties. mass/volume is also an extensive parameter. Here, both assertion and reason are true.

88.

89.

$$W = -P\Delta V = -3\,\text{atm} \times (6-4)\,\text{dm}^3 = -6\,\text{atm L} = -6 \times 101.325\text{J} = -\mathbf{608\ J}$$

90.